DVCS: Reconstruction of $t = \Delta^2$ with Exclusivity Constraint

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(Dated: November 15, 2021)

Imposing exclusivity optimizes the experimental resolution on $t = (q - q')^2$.

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I. INTRODUCTION

In the $e^A Z \to e^A Z\gamma$ reaction, the net 4-momentum transfer to the target is

$$\begin{aligned}
\Delta^\mu = (q-q')^\mu, &\qquad \text{with} \quad q^\mu = (k-k')^\mu \\
k^\mu, \quad k'^\mu = &\ \text{Incident and scattered lepton 4-vectors} \\
q'^\mu = &\ \text{final state photon 4-vector} \\
P^\mu, \quad P'^\mu = &\ \text{Incident and recoil Ion 4-vectors}
\end{aligned} \tag{1}$$

In principle, $\Delta^\mu = (P' - P)^\mu$, but in most cases we cannot measure P' explicitly, but rather infer it from the absence of inelastic production.

In this note, I examine how to optimize the experimental resolution on the reconstruction of $\Delta^2 = (q - q')^2$ from measurements of the scattered electron and photon, while also imposing the exclusivity constraint

$$P'^2 = (P + \Delta)^2 = M_A^2. \tag{2}$$

A key step is to separate Δ^μ into longitudinal and transverse components $\Delta^\mu_{||}$ and $\Delta^\mu_\perp$ such that

$$\begin{aligned}
\Delta^\mu &= \Delta^\mu_\perp + \Delta^\mu_{||} \\
\Delta_\perp \cdot q &= 0 = \Delta_\perp \cdot P, \text{and} \\
\Delta_\perp \cdot \Delta_{||} &= 0 \quad \Rightarrow \quad \Delta^2 = \Delta^2_{||} - \Delta^2_\perp
\end{aligned} \tag{3}$$

It is observed that the resolution on $\Delta^2_\perp$ is generally much better than the resolution on Δ^2, from the measurements of k', q' alone. However, $\Delta^2_{||}$ can be inferred from Q^2, x_{Bj} and the exclusivity condition of Eq. 2. This is then used to improve the resolution of Δ^2.

II. EVENT-BY-EVENT LIGHT CONE VECTORS IN $q \oplus P$ SYSTEM

To define $\Delta^\mu_{||}$ and $\Delta^\mu_\perp$, we start with event-by event light cone vectors n^μ_q, $\widetilde{n}^\mu_q$ that are linear combinations of q and P, and satisfy

$$n_q^2 = 0 = \widetilde{n}_q^2 \qquad \text{and} \quad n_q \cdot \widetilde{n}_q = 1. \tag{4}$$

Define

$$\begin{aligned}
\delta_Q &= \frac{Q^2 M_A^2}{(q\cdot P)^2} \qquad \ll 1 \text{ in DIS kinematics.} \\
n^\mu_q &= \frac{1}{\sqrt{2(1+\delta_Q)}}\left[q^\mu \frac{M_A}{q\cdot P} + P^\mu \frac{\delta_Q}{M_A\left(1+\sqrt{1+\delta_Q}\right)}\right] \\
\widetilde{n}^\mu_q &= \frac{1}{\sqrt{2(1+\delta_Q)}}\left[P^\mu \frac{\left(1+\sqrt{1+\delta_Q}\right)}{M_{\mathrm{A}}} - q^\mu \frac{M_{\mathrm{A}}}{q\cdot P}\right]
\end{aligned} \tag{5}$$

These light cone vectors define the longitudinal and transverse components of Δ^μ:

$$\begin{aligned}
\Delta^\mu_{||} &= (\Delta \cdot n_q)\widetilde{n}^\mu_q + (\Delta\cdot\widetilde{n}_q) n^\mu_q \\
\Delta^\mu_\perp &= \Delta^\mu - \Delta^\mu_{||}
\end{aligned} \tag{6}$$

In this notation, $\Delta_\perp$ is a 4-vector with $\Delta^2_\perp < 0$. We can also define a 3-vector $\vec{\Delta}_\perp$ such that $\vec{\Delta}^2 = -\Delta^2_\perp$.

The light cone components of an arbitrary four-vector Q^μ are defined as

$$A^+ \equiv A \cdot n_q \qquad A^- \equiv A \cdot \widetilde{n}_q. \tag{7}$$

III. EXCLUSIVITY CONSTRAINT

The net invariant momentum transfer in the DVCS reaction is

$$\Delta^2 = 2\Delta^+\Delta^- + \Delta_\perp^2 \tag{8}$$

We now compute $(\Delta \cdot \widetilde{n}_q) = \Delta^-$ not from $(q - q')^\mu$, but rather from Q^2, $q \cdot P$ and exclusivity condition of Eq. 2 (note that by definition of n_q^μ and $\widetilde{n}_q^\mu$, the beam 4-vector P^μ has no transverse components):

$$\begin{aligned} 0 &= \Delta_\perp^2 + 2\Delta^+\Delta^- + 2\Delta^- P^+ + 2\Delta^+ P^- \\ \Delta^-_{\text{Exclusivity}} &= -\frac{\Delta^+ P^- + \Delta_\perp^2/2}{\Delta^+ + P^+} \end{aligned} \tag{9}$$

We then compute an exclusivity corrected value for Δ^2:

$$\begin{aligned} \Delta^2_{\text{Exclusivity}} &= \Delta_\perp^2 + 2\Delta^+\Delta^-_{\text{Exclusivity}} \\ &= \Delta_\perp^2 - \Delta^+ \left[\frac{2\Delta^+ P^- + \Delta_\perp^2}{\Delta^+ + P^+}\right] \end{aligned} \tag{10}$$

Fig. 1 illustrates the reconstruction resolution of Δ^+, Δ^-, $\Delta^-_{\text{Exclusive}}$ and $\Delta^2_{\text{Exclusive}}$.

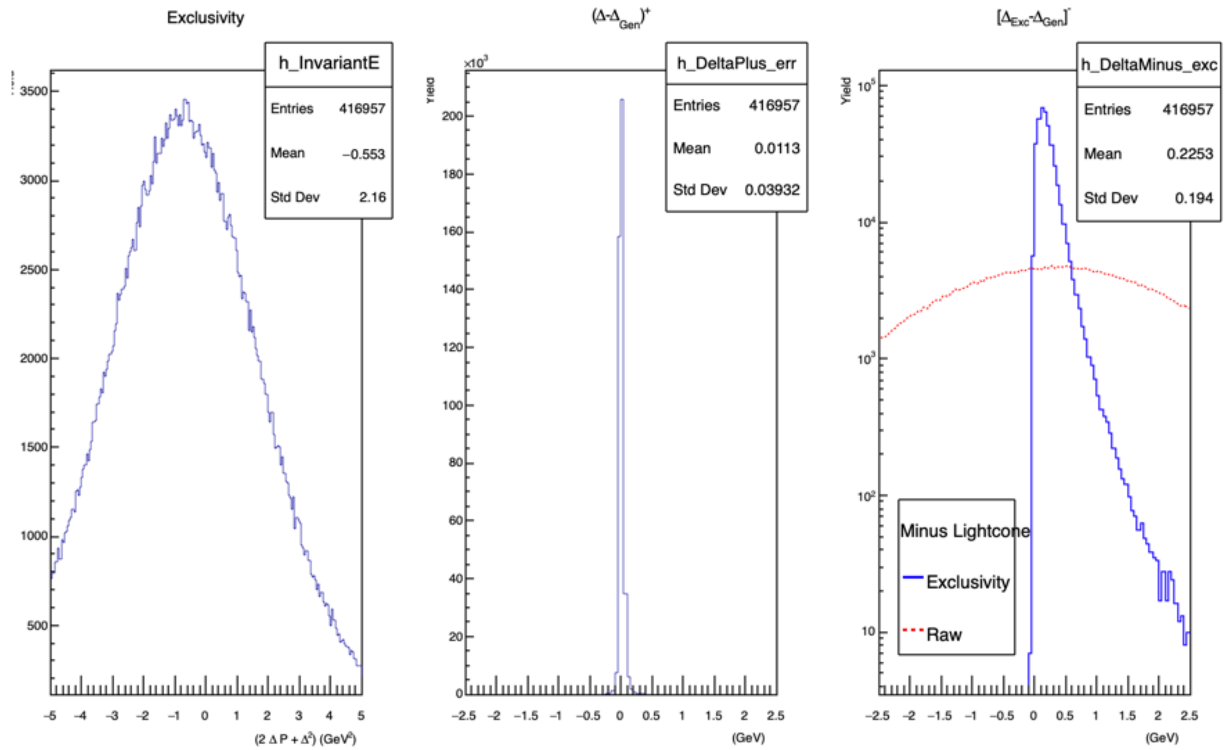

FIG. 1. TOPEG $\alpha(e,e'\gamma)\alpha$ events **Left:** Reconstructed $2\Delta\cdot P+\Delta^2$. **Middle:** Reconstructed $\Delta^+=\Delta\cdot n_q$. **Right:** Dashed red histogram is reconstructed $\Delta^-=\Delta\cdot\widetilde{n}_q$. Solid blue histogram is the recalculated $\Delta^-_{\text{Exclusivity}}$.

Appendix A: Electron Kinematics

Defining

$$\delta_e=\frac{m_e^2M_A^2}{(k\cdot P)^2}\ll 1 \tag{A1}$$

we construct light cone vectors

$$\begin{aligned}
n_e^\mu &= \alpha\left(k^\mu+\frac{m_e^2/(k\cdot P)}{1+\sqrt{1-\delta_e}}P^\mu\right)\\
\widetilde{n}_e^\mu &= \widetilde{\alpha}\left(P^\mu+\frac{M_A^2/(k\cdot P)}{1+\sqrt{1-\delta_e}}k^\mu\right)\\
n_e^2 &= 0=\widetilde{n}_e^2\\
n_e\cdot\widetilde{n}_e &= 1 \qquad \text{provided} \quad \alpha\widetilde{\alpha}=\frac{1+\sqrt{1-\delta_e}}{2(k\cdot P)\sqrt{1+\delta_e}}
\end{aligned} \tag{A2}$$

In this frame, the '$+$' and '$-$'components of a four-vector A^μ are defined by $A^+=A\cdot n$ and $A^-=A\cdot\widetilde{n}$. With this convention in a frame in which the electron and ion are colliding head-on, the incident electron and ion travel in the $-\hat{z}$ and $+\hat{z}$ directions, respectively.

To define Lorentz invariant azimuthal angles, we construct space-like unit vectors X_e^μ, Y_e^μ such that

$$\begin{aligned} X_e^2 = -1 = Y_e^2, &\qquad X_e \cdot Y_e = 0 \\ X_e \cdot n = 0 = X_e \cdot \widetilde{n}, &\qquad Y_e \cdot n = 0 = Y_e \cdot \widetilde{n} \\ \epsilon_{\mu\nu\rho\sigma} \widetilde{n}_e^\mu X_e^\nu Y_e^\rho n_e^\sigma = 1, &\qquad \text{with} \quad \epsilon_{0123} = 1 \end{aligned} \tag{A3}$$

Since the EIC beams will collide nominally in the $x \otimes z$ plane, we start with a reference vector

$$Y_{\rm Det}^\mu = [0, 0, 1, 0]. \tag{A4}$$

Then remove the light cone components and renormalize

$$Y_e^\mu = \left[Y_{\rm Det}^\mu - (Y_{\rm Det} \cdot n)\, \widetilde{n}^\mu - (Y_{\rm Det} \cdot \widetilde{n})\, n^\mu\right] \Big/ \sqrt{1 + 2\,(Y_{\rm Det} \cdot n)\,(Y_{\rm Det} \cdot \widetilde{n})} \tag{A5}$$

The final basis vector is constructed as

$$X_{e,\sigma} = \epsilon_{\mu\nu\rho\sigma} n_e^\mu \widetilde{n}_e^\nu Y_e^\rho \tag{A6}$$

The Lorentz-invariant azimuthal angle of the scattered electron around the collision axis is

$$\Phi_e = \operatorname{atan2}\left(-k' \cdot Y_e, -k' \cdot X_e\right) \tag{A7}$$

Appendix B: Transverse Unit Vectors in the $q \oplus P$ System

The space-like unit vectors in the $q \oplus P$ are defined such that $\mathbf{Y}_q \propto \mathbf{k} \times \mathbf{k}'$. This preserves the Trento Convention.

$$\begin{aligned} Y_q^\mu &= -\sin\Phi_e X_e^\mu + \cos\Phi_e Y_e^\mu \\ X_{q,\sigma} &= \epsilon_{\mu\nu\rho\sigma} n_q^\mu \widetilde{n}_q^\nu Y_q^\rho \end{aligned} \tag{B1}$$

The azimuthal angle $\Phi_{\gamma\gamma} = -\Phi_{NN}$ of the final photon and the transverse momentum transfer squared $\mathbf{\Delta}_\perp^2 = -\Delta_\perp^2$ are defined by

$$\begin{aligned} \sqrt{\mathbf{\Delta}_\perp^2}\cos\Phi_{NN} &= -(k - k' - q') \cdot X_q = -(P' - P) \cdot X_q \\ \sqrt{\mathbf{\Delta}_\perp^2}\sin\Phi_{NN} &= -(k - k' - q') \cdot Y_q = -(P' - P) \cdot Y_q \end{aligned} \tag{B2}$$

Note, however, $(k - k') \cdot X_q = 0 = (k - k') \cdot Y_q$ by construction. Equivalently

$$\begin{aligned} \Phi_{\gamma\gamma} &= \mathrm{atan2}\left(-q' \cdot Y_q, -q' \cdot X_q\right) \\ \boldsymbol{\Delta}_\perp^2 &= \left(q' \cdot X_q\right)^2 + \left(q' \cdot Y_q\right)^2 \end{aligned} \tag{B3}$$

Of particular importance to DVCS, the reconstructed value of $\Delta^2_{ee\gamma}$ is independent of the fluctuations in the initial ion beam momentum. This is not true for Δ^2_{NN}.