Analysis of DVCS Simulations

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The equations defined here are incorporated in my test codes `DVCS_Analysis_v2.cpp` and `CoherentDVCS_MC.cxx`

CONTENTS

I. INTRODUCTION

In this document, I define key variables for testing the DVCS reconstruction performance of an EIC DVCS simulation. The notes include effects of beam emittance and crossing angle.

The DVCS cross section is differential in 5 kinematic variables. I suggest the following invariant set:

$$\left\{Q^2, x_B, \Delta^2 = t, \Phi_e, \Phi_{\gamma\gamma}\right\} \tag{1}$$

The cross section is independent of Φ_e on an unpolarized (or spin-0) ion, but has a non-trivial Φ_e dependence relative to the direction of polarization of a transversely polarized ion. The usual azimuthal angles in the laboratory frame are not intrinsically Lorentz invariant, I will give a proper definition below. It is also useful to define a subsidiary variable

$$\boldsymbol{\Delta}_\perp^2 = -\Delta_\perp^2$$

to be defined below.

Due to the emittance (longitudinal and transverse) of the incident beams, one must also consider the variation of the cross section with $s_e = (q+P)^2$. Thus I suggest to histogram the reconstructed minus generated values of the following seven variables:

$$\left\{s_e, Q^2, x_B, \Delta^2, \Delta_\perp^2, \Phi_e, \Phi_{\gamma\gamma}\right\} \tag{2}$$

II. BEAM PARAMETERS AND EMITTANCE

The nominal incident electron and ion four-momenta are k_0^μ and P_0^μ, respectively. The coordinate system and crossing angle θ_C, ϕ_C are defined as

$$\begin{aligned} k^{0xyz} &= [E_e, 0, 0, -k]\,, \qquad k > 0 \\ P_0 &= [E_A, P_A \sin\theta_C \cos\phi, P_A \sin\theta_C \cos\phi, P_A \cos\theta_C]\,, \qquad P_0^2 = M_A^2 \end{aligned} \tag{3}$$

For IP6, $\theta_C = 25$ mrad and $\phi_C = \pi$. For IP8, $\theta_C = 35$ mrad and $\phi_C = 0$. The event-by-event electron and ion four-momenta are k^μ and P^μ. Fluctuations are assumed to be gaussian, with rms values defined by the beam emittances and the interaction point β-functions as defined in the EIC CDR (table...). Thus the uncertainty in the event-by-event CM energy squared is

$$\Delta s_e = (k+P)^2 - (k_0 + P_0)^2 \tag{4}$$

III. KINEMATIC RECONSTRUCTION

A DVCS simulation, whether parametrized MC or full GEANT4 will produce final state electron, ion, and photon with

Generated four-momenta: $\{k', P', q'\}_{\rm Gen}$,

Reconstructed four-momenta: $\{k', P', q'\}$.

for the scattered electron, 'recoil' ion, and final photon, respectively.

A. DIS and DVCS Invariants

The reconstructed electron DIS variables are defined in relation to the nominal beam parameters as:

$$\begin{aligned} Q^2_{\rm Rec} &= 2k_0 \cdot k' - 2m_e^2 \\ x_{B,\rm Rec} &= \frac{Q^2_{\rm Rec}}{2(k_0 - k')\cdot P_0}\frac{M_A}{M_N} \leq 1 \end{aligned} \tag{5}$$

In contrast, the generated values are based on the event-by-event beam kinematics:

$$\begin{aligned} Q^2_{\rm Gen} &= 2k \cdot k'_{\rm Gen} - 2m_e^2 \\ x_{B,\rm Gen} &= \frac{Q^2_{\rm Gen}}{2(k - k'_{\rm Gen})\cdot P_0}\frac{M_A}{M_N} \end{aligned} \tag{6}$$

There are at least two natural ways to define the momentum transfer four-vector:

$$\Delta^\mu_{e\gamma} = (k - k' - q')^\mu \qquad \text{or} \qquad \Delta^\mu_{NN} = (P' - P)^\mu \tag{7}$$

This leads to the reconstructed and generated invariants

$$\Delta^2_{e\gamma,\rm Rec} = (k_0 - k' - q')^2$$

$$\Delta^2_{NN,\rm Rec} = (P' - P_0)^2 \tag{8}$$

$$\Delta^2_{e\gamma,\rm Gen} = (k_{\rm Gen} - k'_{\rm Gen} - q'_{\rm Gen})^2 = (P'_{\rm Gen} - P_{\rm Gen})^2 = \Delta^2_{NN,\rm Gen} \tag{9}$$

B. Light Cone Vectors, Azimuthal Angles and Transverse Components

I now give invariant definitions of the azimuthal angles and transverse momentum transfer $\Delta^2_\perp$, in a manner consistent with the usual definitions in the target rest frame. To accomplish this, define two sets of light cone vectors, first in the ep frame, and second in the $\gamma^* p$ frame.

1. Electron Kinematics

Defining

$$\delta_e = \frac{m_e^2 M_A^2}{(k\cdot P)^2} \ll 1 \tag{10}$$

we construct light cone vectors

$$
\begin{aligned}
n_e^\mu &= \alpha\left(k^\mu + \frac{m_e^2/(k\cdot P)}{1+\sqrt{1-\delta_e}}P^\mu\right)\\
\widetilde{n}_e^\mu &= \widetilde{\alpha}\left(P^\mu + \frac{M_A^2/(k\cdot P)}{1+\sqrt{1-\delta_e}}k^\mu\right)\\
n_e^2 &= 0 = \widetilde{n}_e^2\\
n_e\cdot\widetilde{n}_e = 1 &\qquad \text{provided} \quad \alpha\widetilde{\alpha} = \frac{1+\sqrt{1-\delta_e}}{2(k\cdot P)\sqrt{1+\delta_e}}
\end{aligned}
\tag{11}
$$

In this frame, the '$+'$ and '$-'$components of a four-vector A^μ are defined by $A^+ = A\cdot n$ and $A^- = A\cdot\widetilde{n}$. With this convention in a frame in which the electron and ion are colliding head-on, the incident electron and ion travel in the $-\hat{z}$ and $+\hat{z}$ directions, respectively.

To define Lorentz invariant azimuthal angles, we construct space-like unit vectors X_e^μ, Y_e^μ such that

$$
\begin{aligned}
X_e^2 = -1 = Y_e^2, &\qquad X_e\cdot Y_e = 0\\
X_e\cdot n = 0 = X_e\cdot\widetilde{n}, &\qquad Y_e\cdot n = 0 = Y_e\cdot\widetilde{n}\\
\epsilon_{\mu\nu\rho\sigma}\widetilde{n}_e^\mu X_e^\nu Y_e^\rho n_e^\sigma = 1, &\qquad \text{with} \quad \epsilon_{0123} = 1
\end{aligned}
\tag{12}
$$

Since the EIC beams will collide nominally in the $x\otimes z$ plane, we start with a reference vector

$$
Y_{\rm Det}^\mu = [0,0,1,0]. \tag{13}
$$

Then remove the light cone components and renormalize

$$
Y_e^\mu = \left[Y_{\rm Det}^\mu - (Y_{\rm Det}\cdot n)\,\widetilde{n}^\mu - (Y_{\rm Det}\cdot\widetilde{n})\,n^\mu\right]\Big/\sqrt{1+2\,(Y_{\rm Det}\cdot n)\,(Y_{\rm Det}\cdot\widetilde{n})} \tag{14}
$$

The final basis vector is constructed as

$$
X_{e,\sigma} = \epsilon_{\mu\nu\rho\sigma}n_e^\mu\widetilde{n}_e^\nu Y_e^\rho \tag{15}
$$

The Lorentz-invariant azimuthal angle of the scattered electron around the collision axis is

$$
\Phi_e = \mathrm{atan2}\left(-k'\cdot Y_e, -k'\cdot X_e\right) \tag{16}
$$

2. DVCS Kinematics

To define the DVCS kinematics, construct a second light cone frame based on (generated) q^μ and P^μ.

$$\begin{aligned}
\delta_Q &= \frac{Q^2 M_A^2}{(q\cdot P)^2} \\
n_q^\mu &= \frac{1}{\sqrt{2(1+\delta_Q)}}\left[q^\mu \frac{M_A}{q\cdot P} + P^\mu \frac{\delta_Q}{M_A\left(1+\sqrt{1+\delta_Q}\right)}\right] \\
\widetilde{n}_q^\mu &= \frac{1}{\sqrt{2(1+\delta_Q)}}\left[P^\mu \frac{\left(1+\sqrt{1+\delta_Q}\right)}{M_\mathrm{A}} - q^\mu \frac{M_\mathrm{A}}{q\cdot P}\right]
\end{aligned} \tag{17}$$

The space-like unit vectors are defined such that $\mathbf{Y}_q \propto \mathbf{k}\times\mathbf{k}'$.

$$\begin{aligned}
Y_q^\mu &= -\sin\Phi_e X_e^\mu + \cos\Phi_e Y_e^\mu \\
X_{q,\sigma} &= \epsilon_{\mu\nu\rho\sigma} n_q^\mu \widetilde{n}_q^\nu Y_q^\rho
\end{aligned} \tag{18}$$

The azimuthal angle $\Phi_{\gamma\gamma} = -\Phi_{NN}$ of the final photon and the transverse momentum transfer squared $\boldsymbol{\Delta}_\perp^2 = -\Delta_\perp^2$ are defined by

$$\begin{aligned}
\sqrt{\boldsymbol{\Delta}_\perp^2}\cos\Phi_{NN} &= -(k-k'-q')\cdot X_q = -(P'-P)\cdot X_q \\
\sqrt{\boldsymbol{\Delta}_\perp^2}\sin\Phi_{NN} &= -(k-k'-q')\cdot Y_q = -(P'-P)\cdot Y_q
\end{aligned} \tag{19}$$

Note, however, $(k-k')\cdot X_q = 0 = (k-k')\cdot Y_q$ buy construction. Equivalently

$$\begin{aligned}
\Phi_{\gamma\gamma} &= \mathrm{atan2}\left(-q'\cdot Y_q, -q'\cdot X_q\right) \\
\boldsymbol{\Delta}_\perp^2 &= \left(q'\cdot X_q\right)^2 + \left(q'\cdot Y_q\right)^2
\end{aligned} \tag{20}$$